SOLVE BY ELIMINATION

$$\boxed{x' = 3y}$$
$$y' = 2x - y$$

$$\longrightarrow \quad x' - 3y = 0, \qquad -2x + y' + y = 0$$

$$\longrightarrow \quad ① \; D[x] - 3[y] = 0$$
$$② \; -2[x] + (D+1)[y] = 0$$

$2[①]$ $\quad 2D[x] - 6[y] = 0$

$D[②]$ $\quad -2D[x] + D(D+1)[y] = 0$

$$(D(D+1) - 6)[y] = 0$$
$$r(r+1) - 6 = 0$$
$$r^2 + r - 6 = 0$$
$$(r+3)(r-2) = 0$$
$$r = -3, 2$$
$$y = c_1 e^{-3t} + c_2 e^{2t}$$

$(D+1)[①]$ $\quad D(D+1)[x] - 3(D+1)[y] = 0$

$3[②]$ $\quad -6[x] + 3(D+1)[y] = 0$

$$(D(D+1) - 6)[x] = 0$$
$$x = k_1 e^{-3t} + k_2 e^{2t}$$

HOW ARE THE COEF'S IN X, Y RELATED?

$$\underline{-3k_1} e^{-3t} + 2k_2 e^{2t} = \underline{3c_1} e^{-3t} + 3c_2 e^{2t}$$

$-3k_1 = 3c_1 \qquad 2k_2 = 3c_2$

$k_1 = -c_1 \qquad k_2 = \frac{3}{2} c_2$

$$\boxed{\begin{aligned} x &= -c_1 e^{-3t} + \tfrac{3}{2} c_2 e^{2t} \\ y &= c_1 e^{-3t} + c_2 e^{2t} \end{aligned}}$$

SOLVE $x' - 3x + 2y = \sin t$
$-4x + y' + y = -\cos t$

$\rightarrow$ ① $(D-3)[x] + 2[y] = \sin t$
② $-4[x] + (D+1)[y] = -\cos t$

$(D+1)[①]$ $\quad (D+1)(D-3)[x] + 2(D+1)[y] = \cos t + \sin t$

$-2\ [②]$ $\quad 8[x] - 2(D+1)[y] = 2\cos t$

$+$

$(D+1)[\sin t] = (\sin t)' + \sin t$
$= \cos t + \sin t$

$((D+1)(D-3)+8)[x] = 3\cos t + \sin t$

$(r+1)(r-3)+8=0$

$r^2 - 2r + 5 = 0 \rightarrow x'' - 2x' + 5x$

$(r^2-2r+1)+4=0$

$(r-1)^2 = -4$

$r-1 = \pm 2i$

$r = 1 \pm 2i$

$x_h = c_1 e^t \cos 2t + c_2 e^t \sin 2t$

$x_p = A\cos t + B\sin t$

$x_p' = B\cos t - A\sin t$

$x_p'' = -A\cos t - B\sin t$

$\cancel{x_p''} - 2x_p' + 5x_p$ $\quad -2B\cos t + 2A\sin t$
$+5A\cos t + 5B\sin t$
$= (4A-2B)\cos t$
$+ (2A+4B)\sin t$
$= 3\cos t + \sin t$

$4A - 2B = 3$

$*2$ $\quad 2A + 4B = 1$

$8A - 4B = 6$

$10A = 7$

$A = \frac{7}{10}$

$*(-2)$

$4A - 2B = 3$

$-4A - 8B = -2$

$-10B = 1$

$B = -\frac{1}{10}$

$x = \frac{7}{10}\cos t - \frac{1}{10}\sin t$
$+ c_1 e^t \cos 2t + c_2 e^t \sin 2t$

$4[①]$ $\quad 4(D-3)[x] + 8[y] = 4\sin t$

$(D-3)[②]$ $\quad -4(D-3)[x] + (D-3)(D+1)[y] = +\sin t + 3\cos t$

$(D-3)[\cos t] = (\cos t)' - 3(\cos t)$
$= +\sin t + 3\cos t$

$$((D-3)(D+1)+8)[y] = \cancel{3}\,5\sin t + 3\cos t$$

$$y_h = k_1 e^t \cos 2t + k_2 e^t \sin 2t$$

$$y_p = C\cos t + E\sin t$$

$4C - 2E = +3$

$2C + 4E = \cancel{3}\,5$

$*2$

$8C - 4E = -6$

$10C = -3$

$C = -\frac{3}{10}$

$*(-2)$

$-4C - 8E = -6$

$4C - 2E = -3$

$-10E = -9$

$E = \frac{9}{10}$

$4C - 2E = 3$

$2C + 4E = 5$

$8C - 4E = 6$

$10C = 11$

$C = \frac{11}{10}$

$4C - 2E = 3$

$-4C - 8E = -10$

$-10E = -7$

$E = \frac{7}{10}$

$$y = \frac{\cancel{-3}\ 11}{10}\cos t + \frac{\cancel{9}\ 7}{10}\sin t + k_1 e^t \cos 2t + k_2 e^t \sin 2t$$

$$x = \frac{7}{10}\cos t - \frac{1}{10}\sin t + c_1 e^t \cos 2t + c_2 e^t \sin 2t$$

$$
\begin{array}{lcl}
-4x & & -\frac{28}{10}\cos t + \frac{4}{10}\sin t - 4c_1e^t\cos 2t - 4c_2e^t\sin 2t \\
+y' & = & \frac{\overset{7}{\cancel{9}}}{10}\cos t + \frac{\overset{-11}{\cancel{3}}}{10}\sin t + k_1e^t\cos 2t - 2k_1e^t\sin 2t \\
 & & \qquad + 2k_2e^t\cos 2t + k_2e^t\sin 2t \quad \Big] \; y' \\
+y & & \frac{\overset{11}{\cancel{-3}}}{10}\cos t + \frac{\overset{7}{\cancel{9}}}{10}\sin t + k_1e^t\cos 2t + k_2e^t\sin 2t \\
\hline
 & & -\cos t \qquad\qquad +
\end{array}
$$